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Do not use any unapproved aids while taking this assessment. Read each question carefully and be sure to show all work in the space provided.

1. Consider the augmented matrix

$$\left[\begin{array}{ccc|c} 0 & 1 & -2 & -3 \\ -1 & 5 & -11 & -13 \\ 0 & 3 & -6 & -9 \\ 1 & -2 & 5 & 4 \end{array}\right]$$

(a) Write a system of scalar equations corresponding to this augmented matrix.

(b) Write a vector equation corresponding to this system.

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2. Consider the vector equation

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -3 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

(a) Write a system of scalar equations corresponding to this augmented matrix.

(b) Write an augmented matrix corresponding to this system.

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3. (a) For each of the following matrices, explain why it is not in reduced row echelon form.

i.
$$A = \begin{bmatrix} 0 & 1 & -1 & -3 \\ 1 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

ii.
$$B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & -7 & 7 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

iii.
$$C = \begin{bmatrix} 1 & -7 & 6 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Show step by step why

$$\text{RREF}\begin{bmatrix} 1 & -3 & 4 & 0 \\ 2 & -5 & 6 & -1 \\ -1 & 2 & -2 & 1 \\ 2 & -5 & 6 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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4. Consider each of the following systems of linear equations or vector equations.

(a)

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 2 \\ 9 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -4 \\ 5 \\ -6 \end{bmatrix} = \begin{bmatrix} -9 \\ -11 \\ 17 \\ -14 \end{bmatrix}$$

i. Explain and demonstrate how to find a simpler system or vector equation that has the same solution set.

ii. Explain whether this solution set has no solutions, one solution, or infinitely-many solutions. If the set is finite, describe it using set notation.

(b)

$$\begin{array}{rcrcrcr} x_1 & + & 3\,x_2 & - & 12\,x_3 & = & 3 \\ -2\,x_1 & - & 5\,x_2 & + & 21\,x_3 & = & -4 \\ x_1 & + & 2\,x_2 & - & 9\,x_3 & = & 1 \\ -3\,x_1 & - & 7\,x_2 & + & 30\,x_3 & = & -5 \end{array}$$

i. Explain and demonstrate how to find a simpler system or vector equation that has the same solution set.

ii. Explain whether this solution set has no solutions, one solution, or infinitely-many solutions. If the set is finite, describe it using set notation.

(c)

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ -4 \\ -2 \\ -19 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -6 \\ -4 \\ -26 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 3 \\ 26 \end{bmatrix}$$

i. Explain and demonstrate how to find a simpler system or vector equation that has the same solution set.

ii. Explain whether this solution set has no solutions, one solution, or infinitely-many solutions. If the set is finite, describe it using set notation.

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5. Consider each of the following systems of linear equations or vector equations.

   (a)

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -4 \\ -3 \\ -8 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 14 \end{bmatrix}$$

   i. Explain and demonstrate how to find a simpler system or vector equation that has the same solution set.

   ii. Explain whether this solution set has no solutions, one solution, or infinitely-many solutions. If the set is finite, describe it using set notation.

   (b)

$$\begin{array}{rcrcrcr} x_1 & & & & & = & 1 \\ 3\,x_1 & + & 4\,x_2 & + & 12\,x_3 & = & 13 \\ & - & x_2 & - & 3\,x_3 & = & -3 \\ 7\,x_1 & + & 7\,x_2 & + & 21\,x_3 & = & 27 \end{array}$$

   i. Explain and demonstrate how to find a simpler system or vector equation that has the same solution set.

   ii. Explain whether this solution set has no solutions, one solution, or infinitely-many solutions. If the set is finite, describe it using set notation.

   (c)

$$\begin{array}{rcrcrcr} x_1 & - & 3\,x_2 & + & 2\,x_3 & = & 4 \\ x_1 & + & 4\,x_2 & - & 5\,x_3 & = & -3 \\ & & 2\,x_2 & - & 2\,x_3 & = & -2 \\ 4\,x_1 & + & 13\,x_2 & - & 17\,x_3 & = & -9 \end{array}$$

   i. Explain and demonstrate how to find a simpler system or vector equation that has the same solution set.

   ii. Explain whether this solution set has no solutions, one solution, or infinitely-many solutions. If the set is finite, describe it using set notation.

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6. Consider the following vector equation.

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ -8 \end{bmatrix}$$

(a) Explain how to find a simpler system or vector equation that has the same solution set.

(b) Explain how to describe this solution set using set notation.

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7. Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2,\ y_1 + y_2)$$

$$c \odot (x, y) = (cx,\ cy - 3\,c + 3)\,.$$

(a) Show that scalar multiplication is associative, that is:

$$a \odot (b \odot (x, y)) = (ab) \odot (x, y).$$

(b) Explain why V nonetheless is not a vector space.

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8. Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 x_2,\ y_1 y_2)$$

$$c \odot (x, y) = (x^c,\ y^c)\,.$$

(a) Show that there exists an additive identity element, that is:

$$\text{There exists } (w, z) \in V \text{ such that } (x, y) \oplus (w, z) = (x, y).$$

(b) Explain why V nonetheless is not a vector space.

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9. Let V be the set of all pairs (x, y) of real numbers together with the following operations:

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2,\ y_1 + y_2)$$

$$c \odot (x, y) = (cx,\ cy - 4\,c + 4)\,.$$

(a) Show that scalar multiplication is associative, that is:

$$a \odot (b \odot (x, y)) = (ab) \odot (x, y).$$

(b) Explain why V nonetheless is not a vector space.

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10. Consider each of these claims about a vector equation.

(a) "$\begin{bmatrix} 4 \\ -5 \\ -2 \\ 2 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 3 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \\ -5 \\ -7 \end{bmatrix}$."

i. Write a statement involving the solutions of a vector equation that's equivalent to this claim.

ii. Determine if the statement you wrote is true or false.

iii. If your statement was true, describe a linear combination of $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 3 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \\ -5 \\ -7 \end{bmatrix}$ that equals $\begin{bmatrix} 4 \\ -5 \\ -2 \\ 2 \end{bmatrix}$.

(b) "$\begin{bmatrix} 3 \\ -4 \\ -1 \\ 1 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 3 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \\ -5 \\ -7 \end{bmatrix}$."

i. Write a statement involving the solutions of a vector equation that's equivalent to this claim.

ii. Determine if the statement you wrote is true or false.

iii. If your statement was true, describe a linear combination of $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 3 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ 0 \\ -5 \\ -7 \end{bmatrix}$ that equals $\begin{bmatrix} 3 \\ -4 \\ -1 \\ 1 \end{bmatrix}$.

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11. Consider each of these claims about a vector equation.

(a) "$\begin{bmatrix} 4 \\ 0 \\ -1 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -5 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 8 \\ 4 \\ -5 \\ -5 \end{bmatrix}$."

i. Write a statement involving the solutions of a vector equation that's equivalent to this claim.

ii. Determine if the statement you wrote is true or false.

iii. If your statement was true, describe a linear combination of $\begin{bmatrix} -5 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 8 \\ 4 \\ -5 \\ -5 \end{bmatrix}$ that equals $\begin{bmatrix} 4 \\ 0 \\ -1 \\ 3 \end{bmatrix}$.

(b) "$\begin{bmatrix} 5 \\ -1 \\ -2 \\ 4 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -5 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 8 \\ 4 \\ -5 \\ -5 \end{bmatrix}$."

i. Write a statement involving the solutions of a vector equation that's equivalent to this claim.

ii. Determine if the statement you wrote is true or false.

iii. If your statement was true, describe a linear combination of $\begin{bmatrix} -5 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 8 \\ 4 \\ -5 \\ -5 \end{bmatrix}$ that equals $\begin{bmatrix} 5 \\ -1 \\ -2 \\ 4 \end{bmatrix}$.