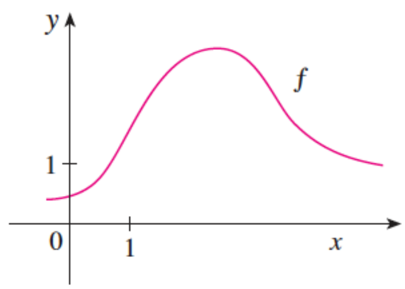
Math 182-Quest 4   April 29, 2022   Name: ______________________________

Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

Question 1: (10pts). Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \sqrt{n} x^n$$

Question 2: (5pts). The graph of f is shown.



Explain why the series

$$1.6 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + \cdots$$

is not the Taylor series of f centered at 1 .

Question 3: (10pts). Evaluate the indefinite integral as an infinite series.

$$\int \frac{e^x - 1}{x} dx$$

Question 4: (10pts). Find a power series representation for the function and determine the interval of convergence.

a) $f(x) = \frac{1}{1+x}$

b) $f(x) = \frac{3}{1-x^4}$

Question 5: (10pts). a) Find the sum of the series. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$

b) Use multiplication of power series to find the first three nonzero terms in the Maclaurin series for each function.

$$y = e^x \ln(1 + x)$$

Question 6: (10pts). True and False

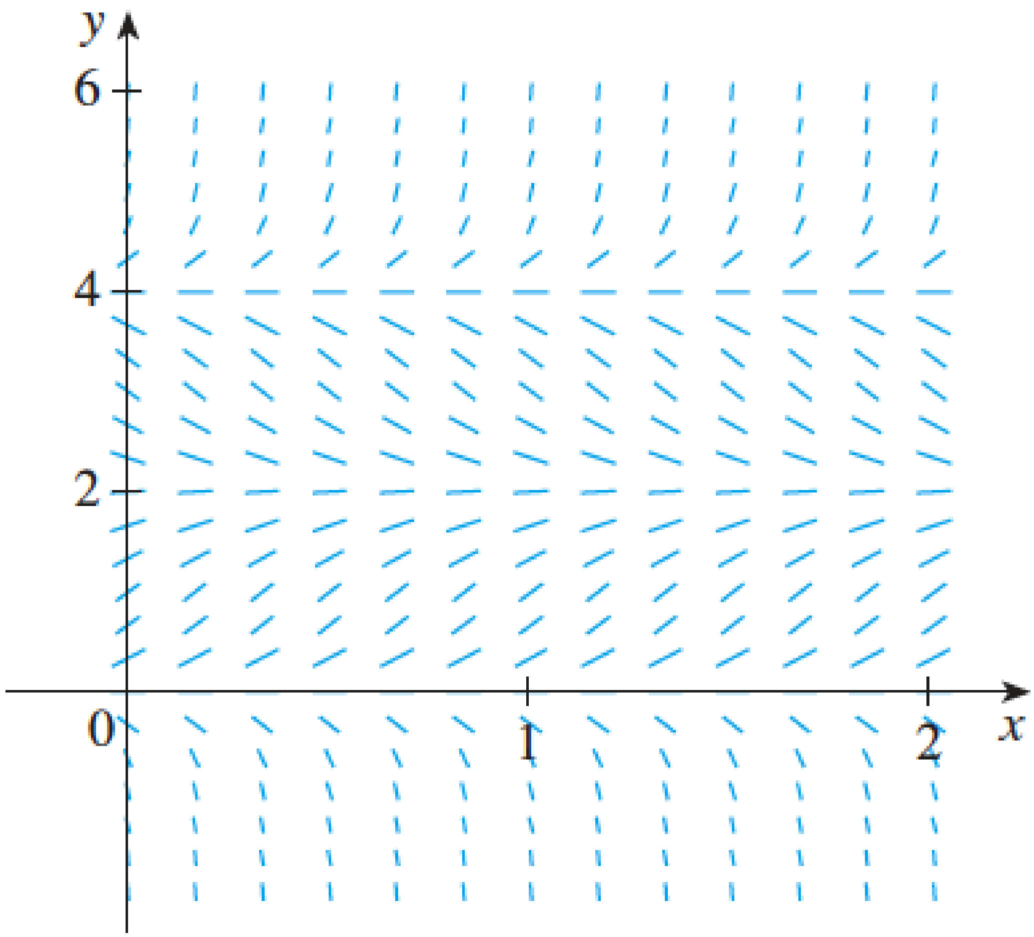
a) The equation $y' = x + y$ is separable.

b) The equation $y' = 3y - 2x + 6xy - 1$ is separable.

Question 7: (10pts). A direction field for the differential equation $y' = y(y-2)(y-4)$ is shown. Sketch the graphs of the solutions that satisfy the given initial conditions.

a) $y(0) = -0.3$

b) $y(0) = 1$



Question 8: (10pts). Which of the following functions are solutions of the differential equation $y'' + y = \sin x$?

a) $y = \sin x$

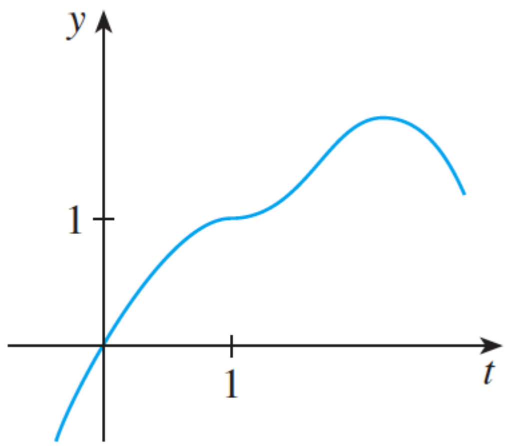
b) $y = \cos x$

c) $y = \frac{1}{2}x\sin x$

d) $y = -\frac{1}{2}x\cos x$

Question 9: (10pts). Explain why the functions with the given graphs can't be solutions of the differential equation

$$\frac{dy}{dt} = e^t(y-1)^2$$



Question Bonus: (10pts). a) Use series to evaluate the following limit.

$$\lim_{x\to 0} \frac{\sin x - x}{x^3}$$

b) The Brentano-Stevens Law in psychology models the way that a subject reacts to a stimulus. It states that if R represents the reaction to an amount S of stimulus, then the relative rates of increase are proportional:

$$\frac{1}{R}\frac{dR}{dt} = \frac{k}{S}\frac{dS}{dt}$$

where k is a positive constant. Find R as a function of S.