Math 181 Sep 17, 2022 Name: _____

Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

Question 1: (10pts). Determine the exact value of the given limit by using algebra to simplify the function.

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$$

Question 2: (15pts). Consider a moving object whose position function is given by $s(t) = t^2$, where s is measured in meters and t is measured in minutes.

a) Determine the most simplified expression for the average velocity of the object on the interval $[3, 3+h]$, where $h > 0$

b) Determine the average velocity of the object on the interval $[3, 3.2]$. Include units on your answer.

c) Determine the instantaneous velocity of the object when $t = 3$. Include units on your answer.

d) Find $s'(3)$.

Question 3: (10pts). Let f be a function with the following properties: f is differentiable at every value of x (that is, f has a derivative at every point),

$$f(-2) = 1, f'(-2) = -2, f'(-1) = -1, f'(0) = 0, f'(1) = 1, f'(2) = 2.$$

Question 4: (10pts). Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leqslant 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

Evaluate each of the following, if it exists.

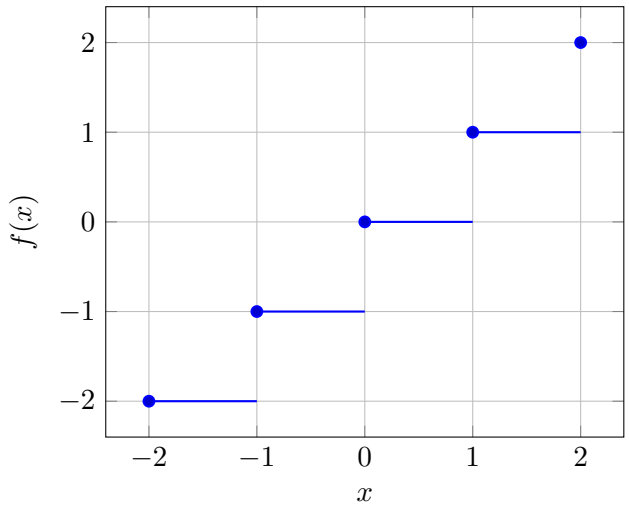
a) $\lim_{x\to 1^-} g(x)$ and $\lim_{x\to 1^+} g(x)$

b) $\lim_{x\to 1} g(x)$

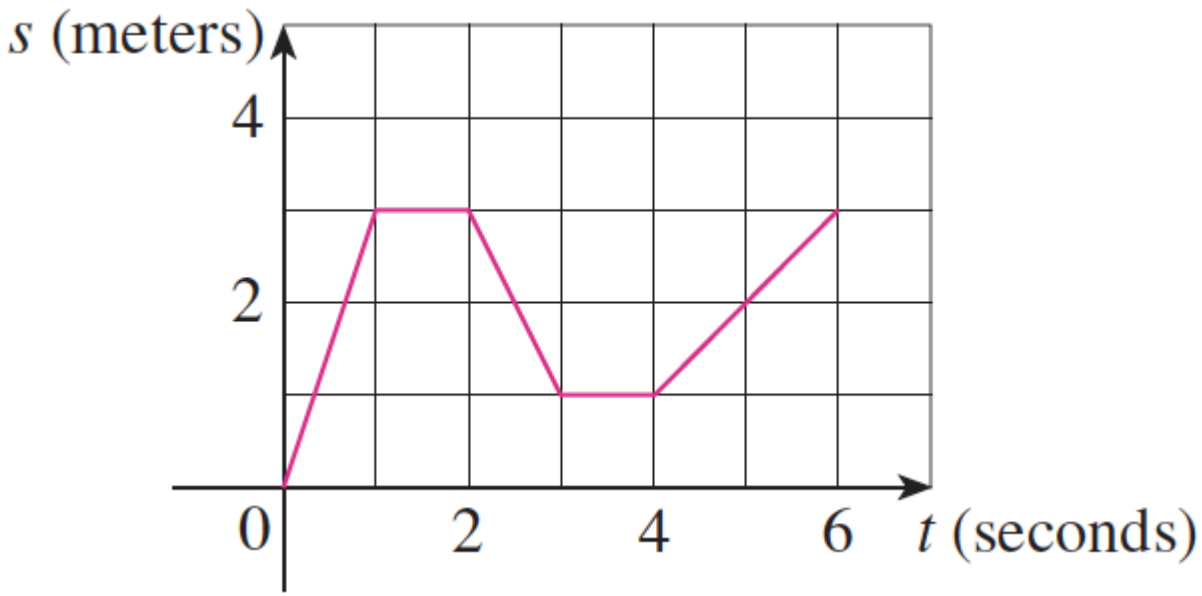
d) $\lim_{x\to 2^-} g(x)$ and $\lim_{x\to 2^+} g(x)$

e) $\lim_{x\to 2} g(x)$

Question 5: (5pts). If the symbol $\lfloor x \rfloor$ denotes the greatest integer function defined evaluate $\lim_{x \to n^-} \lfloor x \rfloor$ and $\lim_{x \to n^+} \lfloor x \rfloor$ For what values of a does $\lim_{x \to a} \lfloor x \rfloor$ exist?



Question 6: (10pts). a) A particle starts by moving to the right along a horizontal line; the graph of its position function is shown. When is the particle moving to the right? Moving to the left? Standing still?



b) Draw a graph of the velocity function.

Question Bonus: (5pts). Is there a number a such that

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

exists? If so, find the value of a and the value of the limit.