Practice Problems Math 181

1. Sketch the graph of a function $h(x)$ that meets all of the following criteria. Be sure to scale your axes and label any important features of your graph.

   - $h(x)$ has a vertical asymptote at 6, but $\lim_{x\to 6^+} h(x)$ is finite.
   - $\lim_{x\to 2} h(x)$ exists and $h(2)$ is defined, but $\lim_{x\to 2} h(x) \neq h(2)$.
   - $\lim_{x\to 5^-} h(x) = 1$ and $\lim_{x\to 5^+} h(x) = 0$.

2. Determine the following limits and explain your reasoning.

   (a)
$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$$

   (b)
$$\lim_{x\to 1} \frac{x^2+2\,x-3}{x^2-6\,x+5}$$

   (c)
$$\lim_{x\to -3} \frac{x^2-8\,x+12}{x^2+11\,x+30}$$

3. (a) Classify the type of discontinuity present at $x=-6$ for the function $f(x)$. Explain your reasoning using limits.
$$f(x) = \begin{cases} -7\,x-50, & x < -6 \\ 5, & x = -6 \\ 3\,x+10, & x > -6 \end{cases}$$

   (b) Determine the value of b to make $h(x)$ continuous at $x=3$. Explain your reasoning using limits.
$$h(x) = \begin{cases} b+\frac{5}{3}\,x, & x \leq 3 \\ -x^2+12\,x-22, & x > 3 \end{cases}$$

4. Explain how to find the value of each limit.

   (a)
$$\lim_{x\to -\infty} -\frac{4\,x^4+5\,x^5-3}{4\,x^3+7\,x^5-1} \quad \text{and} \quad \lim_{x\to +\infty} -\frac{4\,x^4+5\,x^5-3}{4\,x^3+7\,x^5-1}$$

   (b)
$$\lim_{x\to -\infty} \frac{9\,x^4-5\,x^5+7}{9\,x+7\,x^6+1} \quad \text{and} \quad \lim_{x\to +\infty} \frac{9\,x^4-5\,x^5+7}{9\,x+7\,x^6+1}$$

(c)

$$\lim_{x\to-\infty} \frac{x^2+x^4+5}{5\,x+7\,x^3+6} \quad \text{and} \quad \lim_{x\to+\infty} \frac{x^2+x^4+5}{5\,x+7\,x^3+6}$$

5. Explain how to find the value of each limit.

(a)

$$\lim_{x\to 1^-} \frac{(x+6)^2(x+5)^2}{(x-1)^2(x-4)}$$

(b)

$$\lim_{x\to 1^+} \frac{(x+6)^2(x+5)^2}{(x-1)^2(x-4)}$$

(c)

$$\lim_{x\to 1} \frac{(x+6)^2(x+5)^2}{(x-1)^2(x-4)}$$

6. (a) Suppose that there is a function g such that $g(2) = 0.2$ and $g(2.2) = -0.7$. Explain how to use these values to estimate $g'(2)$.

(b) Sketch the graph of a function $f(x)$ that satisfies the following criteria. (You do not need to define the function algebraically.)

- Defined and continuous on the interval $[-10, 10]$.
- $f'(x) > 0$ at $x = 4$
- $\lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = 0$
- The rate of change of $f(x)$ when $x = 2$ is negative
- The graph of $f(x)$ at $x = -5$ has no well-defined tangent line

7. Find $f'(x)$ using the limit definition of the derivative. Then evaluate at $x = -7$.

$$f(x) = \frac{5}{x^3}$$

8. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (scalar multiple, sum/difference, etc.) you are using in your work. (Note that we use the notation $\log(x) = \ln(x) = \log_e(x)$.)

(a)

$$h(y) = \sqrt[4]{y^3} + \frac{7}{y^2}$$

(b)

$$g(w) = 4\,w^5 - 5\,w^2 - 8\,w + 2$$

(c)
$$f(t) = -e^{t} + \sin(t)$$

9. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (product, quotient, sum and difference, etc.) you are using in your work.

(a)
$$h(t) = -\frac{\cos(t)}{2\left(2\,t^{2} + t - 3\right)}$$

(b)
$$f(w) = -2\left(3\,w^{2} + 2\,w + 2\right)e^{w}$$

(c)
$$g(w) = -\frac{4\,w^{2} - 6\,w + 1}{w^{\frac{1}{6}}}$$

10. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (product, quotient, sum and difference, etc.) you are using in your work.

(a)
$$f(t) = 2\,\cos\left(-3\,t^{2} + 2\,t + 3\right)$$

(b)
$$k(x) = \left(2\,x - e^{x} - 2\right)^{4}$$

(c)
$$h(y) = 6\,\sin\left(y^{\frac{2}{3}}\right)$$

(d)
$$g(w) = 6\,\sin(w)^{\frac{2}{3}}$$

11. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (constant multiple, sum and difference, etc.) you are using in your work.

(a)
$$g(w) = \left(2\,w^{4} - 7\,w^{2}\right)^{2} w^{\frac{1}{4}}$$

(b)
$$h(t) = \sqrt{\sin\left(t^{3} - t\right)}$$

(c)

$$f(y) = \left(-\frac{2\left(y^4+1\right)}{y^4-3}\right)^5$$

12. Explain how to use implicit differentiation to find $\frac{dy}{dx}$ for each of the following equations.

(a)

$$6\,x^3 - 8\,y^3 + 2\,e^y + 3 = 0$$

(b)

$$2\,x\cos\left(y\right) = -\sin\left(x\right)$$

13. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (product, quotient, sum and difference, etc.) you are using in your work.

(a)

$$k(x) = \log\left(9\ \arcsin\left(x\right) + 8\ \arctan\left(x\right)\right)$$

(b)

$$n(t) = \frac{\arctan\left(4\,t\right)}{\log\left(t\right)}$$

(c)

$$h(w) = \arcsin\left(w\right)\log\left(w^7-9\right)$$

14. (a) Let $f(x) = x^4+3$. Explain and demonstrate how to find an equation for the line tangent to the graph of $f(x)$ at the point $(-1, 4)$.

(b) Suppose the position of an object in feet is modeled by the following function:

$$s(t) = t^3 + 2\,t^2 - 4\,t + 1.$$

Explain and demonstrate how to find the object's position, velocity, and acceleration at 3 seconds. Use appropriate units for each.

(c) A gadget is sold for $74 per item. Suppose that the number of items produced is equal to the number of items sold and that the cost (in dollars) of producing x gadgets is given by the following function:

$$C(x) = 2\,x^3 + 5\,x^2 + x + 1.$$

Explain and demonstrate how to find the marginal revenue, the marginal cost, and the marginal profit in this situation.

15. Suppose the function $p(x)$ satisfies $p(-8) = -36$, $p'(-8) = 4$, and $p''(x) > 0$ for x values nearby -8.

(a) Explain and demonstrate how to find the linearization $L(x)$ of $p(x)$ at $x = -8$.

(b) Explain and demonstrate how to estimate the value of $p(-7.93)$ using this linearization.

(c) Explain why your estimate of $p(-7.93)$ is greater than or less than the actual value.

(d) Sketch a possible graph of $p(x)$ and its linearization $L(x)$ nearby $x = -8$ to illustrate your findings.

16. Suppose an oil spill in the ocean formed a circle and was growing at a rate of 48 feet^2/minute. When the oil slick reaches a radius of 70, how fast is the radius of the oil spill growing?

17. Explain how to find the global minimum and global maximum values of the function $f(x) = 2\,x^3 - 24\,x + 53$ on the interval $[0, 4]$.

18. Explain how to find the following for the function $f(x) = x^3 + \frac{27}{2}\,x^2 + 54\,x + 7$.

(a) The open intervals where $f(x)$ is increasing or decreasing.

(b) The local extrema of $f(x)$.

19. For each of the following functions, describe the open intervals where it is concave up or concave down, and any inflection points.

(a) $f(x) = -\frac{1}{4}\,x^5 - \frac{5}{3}\,x^4 - \frac{10}{3}\,x^3$

(b) $f(x) = \frac{3}{20}\,x^5 + \frac{1}{4}\,x^4 - 10\,x^3$

20. The following chart describes the values of $f(x)$ and its first and second derivatives at or between a few given values of x, where $\nexists$ denotes that $f(x)$ does not exist at that value of x.

x		-11		-8		-6		-3		-1		1		4		6		9	
$f(x)$		-1		-2		$\nexists$		-1		$\nexists$		-1		$\nexists$		2		1	
$f'(x)$	$-$		$-$		$+$		$-$		$+$		$-$		$-$		$-$		$-$		$-$
$f''(x)$	$-$		$+$		$+$		$+$		$+$		$+$		$-$		$+$		$+$		$+$

Assume that $f(x)$ has vertical asymptotes at each x-value where $f(x)$ does not exist, that $\lim_{x\to-\infty} f(x) = 0$, and that $\lim_{x\to\infty} f(x) = -2$.

Use this information to sketch a reasonable graph of $f(x)$.

21. A farmer currently grows 184 plants, which average a yield of 960 kilograms of product from each plant per year. They could increase or decrease the number of plants grown annually, but due to crowding the yield for each plant would be reduced by 5 kilograms for each additional plant. Explain how the farmer should change their annual growth in order to maximize production.

22. For each limit, explain if L'Hôpital's Rule may be applied. If it can, explain how to use this rule to find the limit.

(a)

$$\lim_{x\to 0} \frac{6\,\sin(7\,x) + 8}{2\,x - 8}$$

(b)

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 7\,x + 10}$$

(c)

$$\lim_{x\to 0} \frac{2\,\cos(3\,x) - 2}{4\,x}$$

(d)

$$\lim_{x\to\infty} \frac{-8\,x^2 - 3\,x + 9}{-5\,x^2 + 3\,x - 7}$$

23. Explain how to use geometric formulas for area to compute the following definite integrals. For each part, sketch the function to support your explanation.

(a)

$$\int_2^{10} \left(-\sqrt{-(x-6)^2 + 16}\right) dx$$

(b)

$$\int_1^6 (-3\,x + 3)\,dx$$

(c)

$$\int_{-1}^2 (-3\,x + 3)\,dx$$

24. Explain and demonstrate how to approximate the area under the curve $f(x) = -7\,x^3 + 2\,x - 6$ on the interval $[-4, 8]$ using a right Riemann sum with 4 rectangles of uniform width.

25. Let $f'(x) = 2\,x - 2$. Find $f(x)$ such that $f(1) = 10$.

26. Explain how to compute the exact value of each of the following definite integrals using the Fundamental Theorem of Calculus. Leave all answers in exact form, with no decimal approximations.

(a)

$$\int_1^5 \left(-2\,e^x\right) dx$$

(b)

$$\int_{\frac{2}{3}\pi}^{\frac{5}{6}\pi} (\sec(x)\tan(x))\,dx$$

(c)
$$\int_{-1}^{1} \left(4\,x^3 + 3\,x^2 - 7\right)\,dx$$

27. Demonstrate and explain how to evaluate the derivative for each of the following definite integrals using the Fundamental Theorem of Calculus.

(a)
$$\frac{d}{dx}\int_{4}^{x} \left(1215\left(2\,e^t - 3\right)^5\right)\,dt$$

(b)
$$\frac{d}{dx}\int_{\cos(x)+7}^{4} \left(4\,\cos\left(t^3 - 6\right)\right)\,dt$$

(c)
$$\frac{d}{dx}\int_{x}^{1} \left(4\,\cos\left(t^3 - 6\right)\right)\,dt$$

(d)
$$\frac{d}{dx}\int_{5}^{e^x+6} \left(5\,e^{\left(t^4+3\right)}\right)\,dt$$

28. Answer the following questions concerning $f(x) = 6\,x^2 + 72\,x + 120$.

(a) What is the total area between $f(x) = 6\,x^2 + 72\,x + 120$ and the x-axis from $x = -7$ to $x = 1$?

(b) What is the net area between $f(x) = 6\,x^2 + 72\,x + 120$ and the x-axis from $x = -7$ to $x = 1$?