Name: _____

Math 181 -Final Exam M#: _____

Do not use any unapproved aids while taking this assessment. Read each question carefully and be sure to show all work in the space provided.

1. Sketch the graph of a function $g(x)$ that meets all of the following criteria. Be sure to scale your axes and label any important features of your graph.

   - $\lim_{x\to -3^-} g(x)$ and $\lim_{x\to -3^+} g(x)$ are both infinite.
   - $\lim_{x\to -4} g(x)$ exists and $g(-4)$ is defined, but $\lim_{x\to -4} g(x) \neq g(-4)$.
   - $\lim_{x\to -5^-} g(x) \neq \lim_{x\to -5^+} g(x)$.

2. Determine the value of b to make $f(x)$ continuous at $x = 1$. Explain your reasoning using limits.

$$f(x) = \begin{cases} -5\,x^2 + b + 3\,x, & x \leq 1 \\ x^2 + \frac{1}{2}\,x - \frac{1}{2}, & x > 1 \end{cases}$$

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3. (a) Suppose that there is a function g such that $g(-1) = -0.2$ and $g(-0.9) = 0.5$. Explain how to use these values to estimate $g'(-1)$.

(b) Sketch the graph of a function $f(x)$ that satisfies the following criteria. (You do not need to define the function algebraically.)

- Defined and continuous on the interval $[-10, 10]$.
- The slope tangent to the graph of $f(x)$ at $x = -6$ is positive
- $\lim_{h \to 0} \dfrac{f(4+h) - f(4)}{h} = 0$
- $f'(x) < 0$ at $x = 6$
- The rate of change of $f(x)$ when $x = 1$ is not well-defined

4. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (product, quotient, sum and difference, etc.) you are using in your work.

(a)
$$k(w) = \log\left(6 \arcsin(w) + 4 \arctan(w)\right)$$

(b)
$$h(t) = -2 \arcsin(t) \log\left(t^2 + 9\right)$$

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5. Suppose the function $p(x)$ satisfies $p(3) = -23$, $p'(3) = -7$, and $p''(x) > 0$ for x values nearby 3.

   (a) Explain and demonstrate how to find the linearization $L(x)$ of $p(x)$ at $x = 3$.

   (b) Explain and demonstrate how to estimate the value of $p(3.07)$ using this linearization.

   (c) Explain why your estimate of $p(3.07)$ is greater than or less than the actual value.

6. Suppose a ladder of length 22 ft is leaning against a wall. Then the base of the ladder is pulled away from the way at a rate of 5 feet/second. What is the velocity of the top of the ladder when the base of the ladder is 12 feet away from the wall?

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7. Explain how to find the global minimum and global maximum values of the function $f(x) = 2\,x^3 + 48\,x^2 + 330\,x - 34$ on the interval $[-10, -3]$.

8. The following chart describes the values of $f(x)$ and its first and second derivatives at or between a few given values of x, where $\nexists$ denotes that $f(x)$ does not exist at that value of x.

x	-10		-8		-6		-3		0		2		5		8		10	
$f(x)$	-1		1		$\nexists$		-3		$\nexists$		-2		$\nexists$		-2		0	
$f'(x)$	$+$	$+$		$+$		$+$		$-$		$-$		$-$		$-$		$+$		$+$
$f''(x)$	$+$	$+$		$+$		$-$		$-$		$+$		$-$		$+$		$+$		$-$

Assume that $f(x)$ has vertical asymptotes at each x-value where $f(x)$ does not exist, that $\lim_{x\to-\infty} f(x) = -3$, and that $\lim_{x\to\infty} f(x) = 1$.

Use this information to sketch a reasonable graph of $f(x)$.

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9. You've been asked to design a metalic box that is constructed from a rectangular sheet of metal of size 528×528 inches^2. Four equally sized squares must be cut from each corner of the rectangle, so that the remaining material can be folded and welded into a lidless box. How large should these squares be to maximize the volume that this box can contain?

10. For each limit, explain if L'Hôpital's Rule may be applied. If it can, explain how to use this rule to find the limit.

    (a)
    $$\lim_{x \to \infty} \frac{2\,x^2 + 9\,x - 2}{-8\,x^2 - 6\,x + 8}$$

    (b)
    $$\lim_{x \to 0} \frac{-5\,\sin(6\,x) + 3}{2\,x + 7}$$

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11. Explain how to use geometric formulas for area to compute the following definite integrals. For each part, sketch the function to support your explanation.

(a)

$$\int_{-1}^{2} (-4\,x + 4)\,dx$$

(b)

$$\int_{4}^{8} \left(-\sqrt{-(x-6)^2 + 4}\right) dx$$

12. Let $f'(x) = -6\,x^2 + 1$. Find $f(x)$ such that $f(-3) = 52$.

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13. Explain how to compute the exact value of each of the following definite integrals using the Fundamental Theorem of Calculus. Leave all answers in exact form, with no decimal approximations.

(a)
$$\int_3^0 \left(-2\,x^3 + 6\,x + 6\right) dx$$

(b)
$$\int_{\frac{7}{6}\pi}^{\frac{5}{4}\pi} \left(6\,\sec\left(x\right)\tan\left(x\right)\right) dx$$

(c)
$$\int_4^4 \left(5\,e^{x^2}\right) dx$$

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14. (a) Demonstrate and explain how to evaluate the derivative for each of the following definite integrals using the Fundamental Theorem of Calculus.

$$\frac{d}{dx}\int_{5}^{\sin(x)+1}\left(7\left(8\cos\left(t\right)+7\right)^{5}\right)dt$$

(b) What is the total area between $f(x)=6\,x^2+12\,x-48$ and the x-axis from $x=-1$ to $x=4$?